A Combined Scheme of Adiabatic Compression and Accelerated Merging of Tokamak Tori

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Abstract

It is shown that inclusion in the analysis of the surface current induced during rapid adiabatic compression resolves the main discrepancy between the simple scaling laws of Furth-Yoshikawa and detailed numerical calculations. A scheme combining adiabatic compression and accelerated merging, producing no net surface current, is suggested.

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1. Introduction

Adiabatic compression along the major radius is a simple way to increase the temperature of tokamak plasmas. However, from an economic point of view, this heating method is not likely to be viable in future fusion power plants because of its pulse type work in nature and, after being compressed to a much smaller major radius, only a small portion of the toroidal magnetic energy can be utilized. Nevertheless, from the experimental point of view, adiabatic compression may be an inexpensive way to reach the temperature that must be attained in the experimental ignition facilities.

Furth and Yoshikawa[1] derived some simple laws for adiabatic compression several years ago. Recently, however, more detailed calculations have shown that large discrepencies exist between numerically calculated toroidal current and poloidal beta [2,3,4] and those predicted by the Furth-Yoshikawa scaling laws. It is one of the purposes of this paper to attempt to resolve the discrepancies. Some authors[2,3] have claimed that the difference is due to the assumption of large aspect ratio in the Furth-Yoshikawa theory. However, we will show that the discrepanices can be resolved unambiguously if we take into account the induced surface current which was neglected in the analysis of Furth and Yoshikawa. In this regard, it will also be shown that the assumption of large aspect ratio plays an insignificant role. It will be shown in this paper that the direction of the surface toroidal current is in the same direction as that of the main toroidal current, this surface current may degrade the stability properties of toroidal plasmas. Albert[3] suggested regulating the poloidal flux of the Ohmic heating coils accordingly to compensate for the surface current. It is doubtful that this method could be used practically , since the time constant of the Ohmic heating coils is much larger than that of the vertical field coils.

Recently another method[5,6] to increase the plasma temperature has been suggested. Principally, there is an electromagnetic attraction force between two separated tori, with the toroidal currents in the same direction. During the acceleration process, part of the poloidal magnetic energy is converted to the kinetic energy of the plasmas and is subsequently dissipated as thermal energy by collision. Recent experiments have given encouraging results. In comparison with adiabatic compression, this method allows simplification of the design and arrangement of the toroidal coils. One possible problem that might exist is that a surface current in the direction opposite to that of the main current is also induced during the accelerating period; this may also degrade. the stability properties of plasmas.

A new scheme of heating is suggested, combining adiabatic compression along the major radius and axial acceleration of the tori. Since the induced surface currents are in the opposite direction when these two processes work separately, it is possible that the surface current can be cancelled when they work simultaneously under certain conditions ,without regulating the Ohmic heating coils.

In order to get a clear physical picture of this process, we analyze pure radial adiabatic compression and pure axial acceleration. We then consider the combined scheme.

II. Pure Radial Adiabatic Compression

Equilibrium of toroidal plasma needs an applied vertical field, and the total poloidal magnetic field is the combination of the induced poloidal field of the plasma and the vertical field. In a low beta plasma, the induced toroidal field is much smaller than the applied toroidal field, so it can be neglected without introducing much inaccuracy.

If a surface toroidal current is induced in an infinite conducting plasma, it does not change the magnetic structure inside the plasma directly, but it does change the poloidal magnetic field outside the plasma torus. Since the total toroidal current has a step change across the plasma boundary, the β_p and q also have a step change across the boundary. Now we can divide the total toroidal plasma current into two parts: one is the volume current I_{tv} and the other is the surface current I_{ts}

$$I_t = I_{tv} + I_{ts}. \tag{1}$$

During the adiabatic compression process of ideal plasma, two conditions must be satisfied within the plasma:

$$a^2 B_t = \text{const.} \tag{2}$$
$$q(a_-) = \text{const.} \quad (q(\psi) = \text{const.}). \tag{3}$$

$a_{+,-}$ means the minor radius just above or below the plasma surface. From these two conservation conditions, we can derive the Furth-Yoshikawa scaling laws (with $B_t R = \text{const.}$).

$$\text{from(2)} \qquad B_t \infty a^{-2} \tag{4}$$
$$\text{from(3)} \qquad q(a) = \frac{a_- B_t}{R B_p(a)} \infty \frac{a_- B_t}{R I_{tv}/a} = \text{const.} \tag{5}$$
$$\text{from(5)} \qquad I_{tv} \infty \frac{1}{R} \tag{6}$$

the last equation shows that the volume toroidal current is inversely proportional to the major radius R. But (2) and (3) are not the only two conditions that must be satisfied during adiabatic compression; the total poloidal magnetic flux outside the torus must also be satisfied during adiabatic compression.

$$\psi_{\text{out}} = \frac{4\pi}{c} R(ln 8\frac{R}{a} - 2) I_t - \int_0^{R-a} B_\perp 2\pi r dr = \text{const.} \tag{7}$$

and from the equilibrium condition

$$B_{\perp} = \frac{I_t}{cR}(ln8\frac{R}{a} + \beta_p - \frac{3}{2} + \frac{li}{2}) \tag{8}$$

Here we treated two limiting cases:

Case I: $B_{\perp}$ is homgeneous in the whole region

$$B_{\perp} \neq f(r)$$

then Eq. (7) can be integrated to

$$\psi_{\text{out}} = \frac{4\pi}{c}R(ln8\frac{R}{a} - 2)I_t - \frac{I_t}{cR}(ln8\frac{R}{a} + \beta_p - \frac{3}{2} + \frac{li}{2})\pi(R-a)^2 = \text{const.} \tag{9}$$

so

$$\frac{I_{t2}}{I_{t1}} = \frac{R_1}{R_2}\frac{[ln8R_1/a_1 - 2 - 1/4(ln8R_1/a_1 - 3/2 + \beta_{p1}(a_+) + l_{i1}/2)(1 - a_1/R_1)^2]}{[ln8R_2/a_2 - 2 - 1/4(ln8R_2/a_2 - 3/2 + \beta_{p2}(a_+) + l_{i2}/2)(1 - a_2/R_2)^2]} \tag{10}$$

where

$$\beta_p(a_+) = \frac{\langle p \rangle}{B_p^2(a_+)/8\pi}$$

Case II: $B_{\perp} = 0$ outside the plasma

$$\psi_{\text{out}} = \frac{4\pi}{c}I_t R(ln8\frac{R}{a} - 2) = \text{const.} \tag{11}$$

$$\frac{I_{t2}}{I_{t1}} = \frac{R_1}{R_2}(\frac{ln8R_1/a_1 - 2}{ln8R_2/a_2 - 2})$$

The difference between I_{t2} and I_{tv_2} is the total surface current I_{ts_2}

$$I_{ts_2} = I_{t2} - I_{tv_2}.$$

It is clear from the above analysis that Furth-Yoshikawa had only considered the I_{tv_2} without considering the existence of I_{ts_2}. It is also shown that the $q(a_-)$ is conserved during the rapid adiabatic compression, but $q(a_+)$ is not conserved. Since a_+ is already in the vacuum region, there is no reason why $q(a_+)$ must be conserved.

The real case is between these two limiting cases. It does show a large deviation from the Furth-Yoshikawa scaling law on the total toroidal current and poloidal beta due to the existence of surface current whether or not the aspect ratio is large enough (Fig. 1).

III. Pure Axial Acceleration and Merging

The total magnetic energy of two mutually interacting tori is

$$W = 2(\frac{1}{2}\frac{L}{c^2}I_t^2) + \frac{M}{c^2}I_t^2 \tag{12}$$

where L, I_t is the inductance and toroidal current of one torus, and M is the mutual inductence.

The force along the Z direction is

$$F_z = \frac{\partial W}{\partial z}|_{I_t} = \frac{I_t^2}{c^2}\frac{\partial M}{\partial z}, \tag{13}$$

and the equation of momentum is

$$M_p\frac{dV_z}{dt} = M_pV_z\frac{dV_z}{dz} = \frac{I_t^2}{c^2}\frac{\partial M}{\partial z} \tag{14}$$

where $M_p = 2 \times \langle n \rangle m_i \pi a^2 \times 2\pi R_1$ since magnetic flux is conserved during acceleration. That is,

$$(L + M)I_t = \text{const.}. \tag{15}$$

Substituting (15) into (14) and integrating with respect to z

$$M_p\frac{V^2}{2} = \frac{I_{t1}^2}{c^2}(L + M_1)^2(\frac{1}{L + M_1} - \frac{1}{L + M}) \tag{16}$$

As a rough estimate, we assumed $M_1 \approx 0$ for two initially well-separated tori and $M_2 \approx L$ when the two tori are next to each other. Then from (16)

$$M_p\frac{V_2^2}{2} \approx \frac{I_{t1}^2}{2c^2}, \tag{17}$$

For example, two well-separated tori with parameters near Alcator A, the final velocity is

$$V_2 \approx (\frac{I_{t1}^2}{c^2}\frac{L}{M_p})^{1/2}$$

with

$$\begin{aligned} M_p &= 2 \times \langle n \rangle m_i \pi a^2 \times 2\pi R \\ &= 2 \times 2 \times 10^{14} \times 1.67 \times 10^{-24} \times \pi(9.5)^2 \times 2\pi \times 54 \\ &= 6.4 \times 10^{-5} g \\ I_{t1} &= 10^5 A \end{aligned}$$

$$V_2 \approx 0.54 \times 10^8 \text{cm/sec}.$$

and

$$\frac{I_{t2}}{I_{t1}} = \frac{L+M_1}{L+M_2} \approx \frac{1}{2} \qquad (\text{if} \frac{M_1}{L} \ll 1) \tag{18}$$

since $q(a\_)$ is conserved, so the volume current is conserved. If the total current decreases to half of its initial value, then a surface current must be induced in the opposite direction of the main electric current. The change of the poloidal magnetic energy during the acceleration process is

$$\begin{aligned} \Delta W_p &= 2(\frac{1}{2}\frac{L}{c^2}I_{t1}^2) + MI_{t1}^2 - \frac{1}{2}LI_{t1}^2 \\ &\approx \frac{1}{2c^2}LI_{t1}^2 \end{aligned} \tag{19}$$

and it is consistent with the expression (18). The total initial thermal energy is

$$W_{th} = 12\pi^2 a^2 RnkT. \tag{20}$$

sothat

$$\frac{\Delta W_p}{W_{th}} = \frac{1}{\beta_p}\frac{4}{6}(ln8\frac{R}{a} - 2 + \frac{li}{2}) \approx \frac{1}{\beta_p}. \tag{21}$$

Axial acceleration and merging seem to be an efficient heating method only for low β_p and not for high β_p tokamak. The poloidal magnetic energy that can be released to heat the plasma is quite limited.

IV. Combined Scheme of Radial Compression and Axial Acceleration

Since pure radial compression and pure axial acceleration induce the surface currents in the opposite direction, it is natural to consider combining these two processes. Under certain coditions, it is possible to obatin a dynamic process without inducing surface currents.

Fig. (2) illustrates two plasma tori being compressed and accelerated along paths making an angle α with the horizontal direction. The following conditions are still satisfied within the plasma.

$$B_t a^2 = \text{const.} \tag{22a}$$

$$B_t R = \text{const.} \tag{22b}$$

$$q(a\_) = \text{const.}. \tag{22c}$$

From these three conditions, we obtain

$$a \infty R^{1/2} \tag{23a}$$

$$I_{tV} \infty \frac{1}{R}. \tag{23b}$$

Although it is not difficult to introduce any distribution of the vertical field outside the plasma, we consider here a simple case where the vertical field is zero outside the plasma.

Another condition that must be conversed is

$$\Psi_{\text{out}} = (L + M) I_t = \text{const.}. \tag{24a}$$

That is,

$$\frac{dI_t}{I_t} = -\frac{d(L + M)}{L + M} \tag{24b}$$

from (23b) we obtain

$$\frac{dI_{tV}}{I_{tV}} = \frac{-dR}{R} \tag{24c}$$

since what we need is $I_{ts} = 0, I_t \equiv I_{tV}$; that is to say, no surface current is induced, so

$$\frac{dI_t}{I_t} = \frac{dI_{tV}}{I_{tV}} \tag{25}$$

put (24a), (23c) and (25) together

$$\begin{aligned} \frac{dR}{R} &= \frac{dL}{L+M} + \frac{dM}{L+M} \\ &= \frac{dL}{L+M} + \frac{\partial M/\partial R dR + \partial M/\partial z dz}{L+M} \\ tg a = \frac{dz}{dR} &= \frac{L + M/R - dL/dR - \partial M/\partial R}{\partial M/\partial z} \end{aligned} \tag{26}$$

where

$$
\begin{aligned}
L &= 4\pi R(ln8\frac{R}{a} - 2) \\
M &= \frac{8\pi R}{R}[(1 - \frac{1}{2}k^2)K - E] \\
k^2 &= \frac{R^2}{R^2 + z^2} \\
K &= \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin\theta}} \\
E &= \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta d\theta} \\
\frac{\partial M}{\partial R} &= \frac{2\pi R}{\sqrt{R^2 + z^2}}[K - E] \\
\frac{\partial M}{\partial z} &= -\frac{4\pi z}{\sqrt{R^2 + z^2}}[-K + \frac{R^2 + 2z^2}{2z^2}E]
\end{aligned}
$$

The real path is the simple numerical integration of Eq. (26), as an initial value problem, for illustration we choose a typical value ($z/R = 0.5$) to estimate the local angle α

$$\alpha|_{z/R=0.5} = \mathrm{tg}^{-1}\, 1.03 \approx 46°$$

V. Conclusions

1. Rapid major radius adiabatic compression induces a surface current on the torus, which Furth-Yoshikawa previously neglected. It is this surface current, and not the assumption of large aspect ratios, that causes the main discrepancy between the simple scaling laws and more detailed numerical calculations.
2. Axial acceleration with merging does not seem to be a convenient heating method for high β_p ($\beta_p \geq 1$) plasmas.
3. A combined scheme of radial compression and axial acceleration is suggested. The surface current could be completely cancelled under certain conditions. The typical compression angle is $\alpha \approx 45°$.

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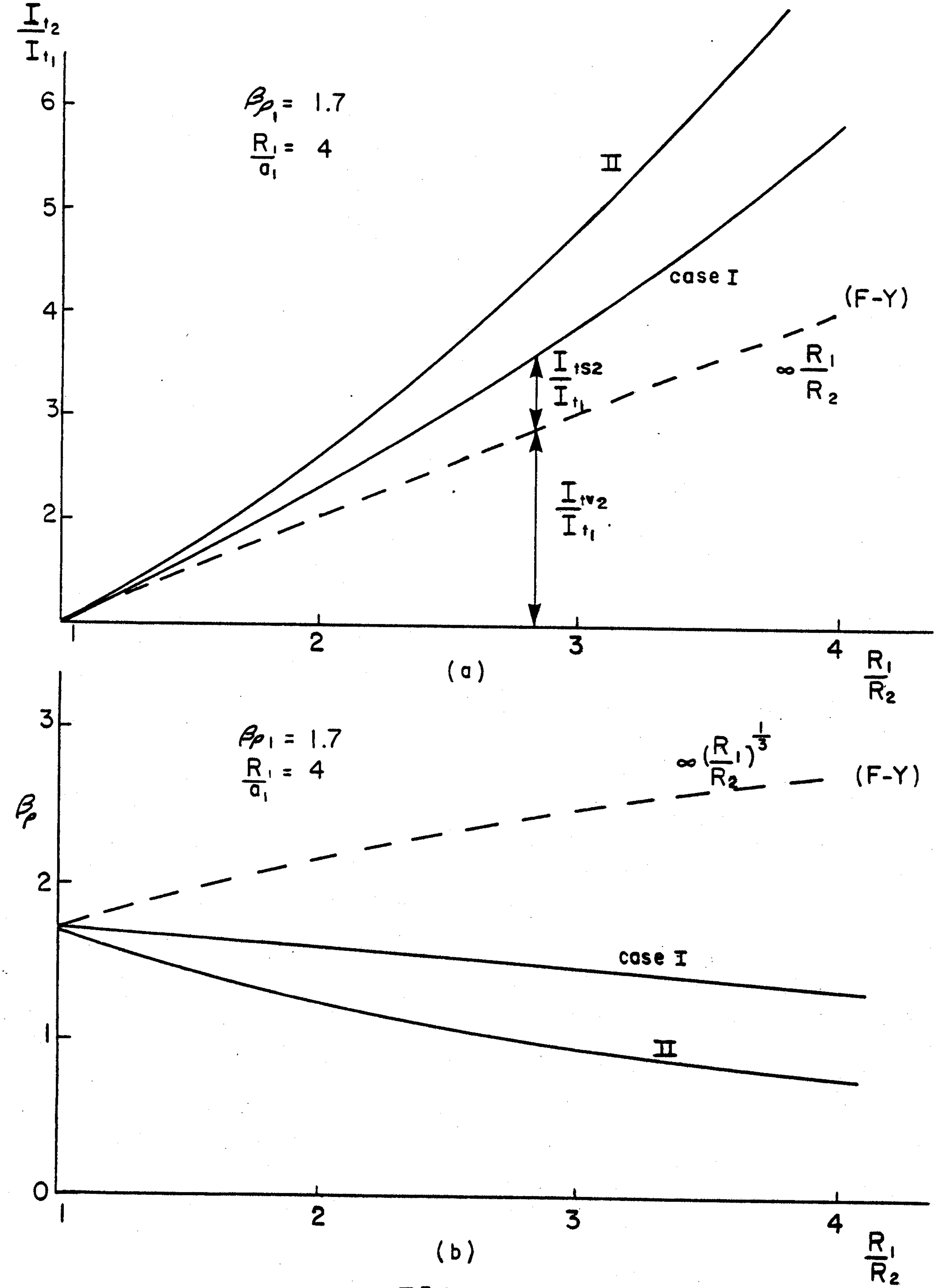
FIGURE CAPTIONS

Fig. 1. The variation of total toroidal current (a) and poloidal beta (b) as compared with simple laws predicted by Furth-Yoshikawa's theory. Case I: Vertical field is homogenous. Case II: Vertical field is absent outside the torus.

Fig. 2. Combined Scheme of Radial compression and Axial Acceleration without inducing surface current.



$\frac{I_{t_2}}{I_{t_1}}$
6
5
4
3
2
$\beta_{\rho_1} = 1.7$
$\frac{R_1}{a_1} = 4$
II
case I
(F-Y)
$\frac{I_{ts2}}{I_{t_1}}$
$\propto \frac{R_1}{R_2}$
$\frac{I_{tv2}}{I_{t_1}}$
1
2
3
4
$\frac{R_1}{R_2}$
(a)
β_ρ
3
2
1
0
$\beta_{\rho_1} = 1.7$
$\frac{R_1}{a_1} = 4$
$\propto \left(\frac{R_1}{R_2}\right)^{\frac{1}{3}}$
(F-Y)
case I
II
1
2
3
4
$\frac{R_1}{R_2}$
(b)

FIG. 1

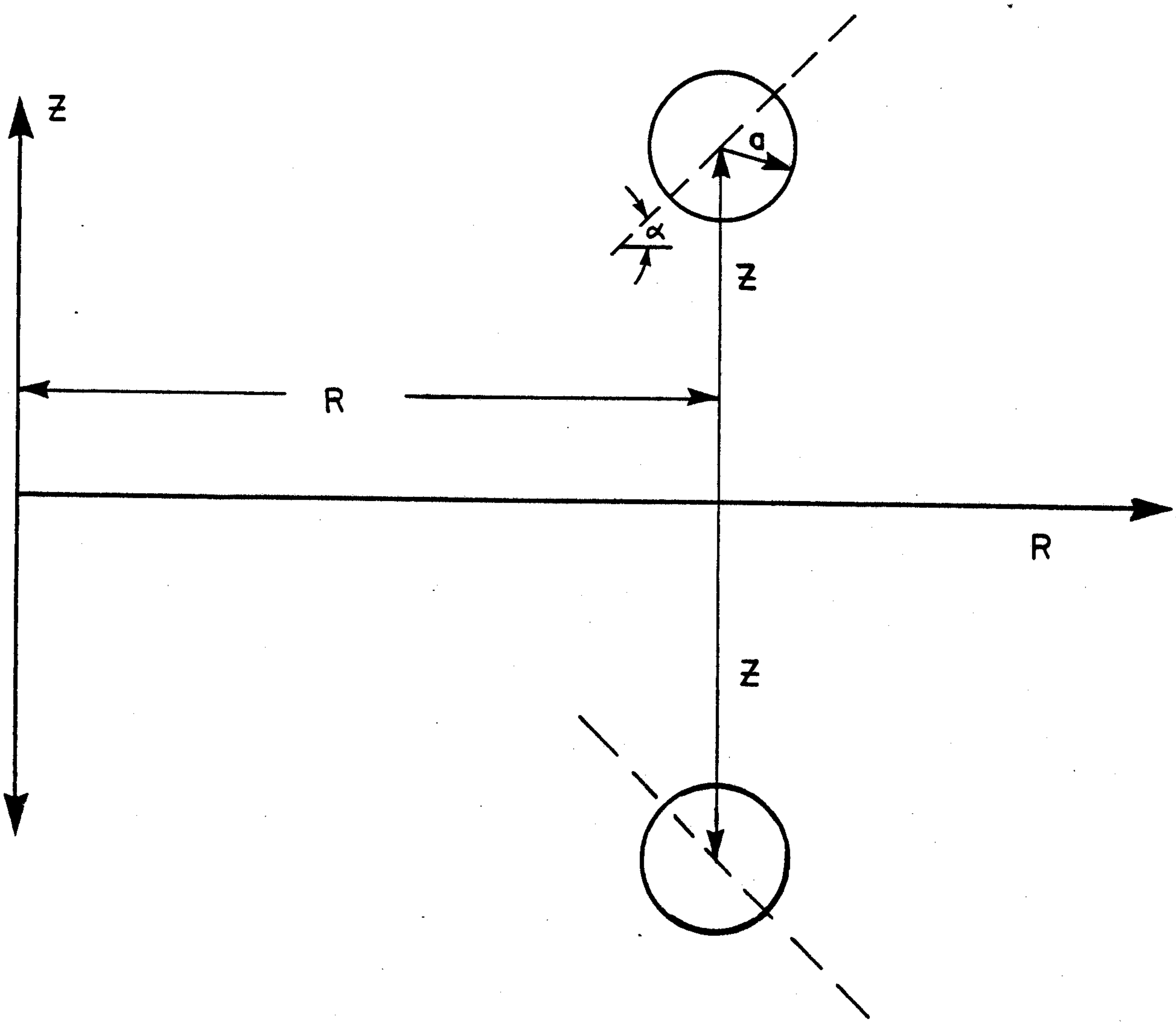


FIG. 2